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SS-MODULES

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ABSTRACT

Let R be a commutative ring with identity and let M be a unitary R-module. In this paper, we introduce the concept of SS-Modules some properties and characterizations of SS-Modules are given.Also, various basic results about SS-Modules and regular modules are considered.

KEYWORDS: SS-Modules, Finitely Generated Module, Regular Ring and Regular Module

1. INTRODUCTION

Every ring considered in this paper will be assumed to be commutative with identity and every module is unitary. We introduce the following :- An R-module M is called SS-Module if and only if annRM is a semimaximal ideal of R,where $ann_R M$ = {r: r∈R and Rm =0 for all m∈M}, [1].

Our concern in this paper is to study SS-Module and look for any realationbtween SS-Module and certain type of well-known modules specially with semiprimemoduls.

This paper consists of two sections. Our main concern in section one, is to define and study SS-Modules, and we give some characterizations for this concept.In section two, we study the relation between SS-Modules and regular modules.

2. SS-MODULES

Definition (2.1)

A non-zero R-module is called SS-Module if and only if $ann_R M$ is semimaximal ideal of R.

Remarks and Examples (2.2)

(1) Every maximal ideal is semimaximal ideal, but the converse is not true in general, for example: - 6Z are a semimaximal ideal of a ring Z which is not maximal, see [2].

(2) Z_6 as a Z—module is SS-Module, since annZ (Z_6) =6Z is semimaximal of Z.

(3) Z_{10} as a Z-module is SS-Module, since $ann_Z(Z_{10})$ =10Z is semimaximal of Z.

(4) Condider M=$\oplus PZ_P$ as a Z-module is not SS-Module. In fact $ann(\oplus pZ_P) = \cap_P (ann(Z_P) = \cap_P (PZ) = (0)$and $(0)$is not semimaximal ideal of Z.

(5) For each positive integer n, the Z-module Z⊕Zn is not SS-Module, since ann_z(Z⊕Zn) = (0) is not semimaximal ideal of Z.

(6) Z as a Z-module is not SS-Module.

(7) Every submodule of the SS - Module is SS-Module.

Proof

Let N be a non-zero proper submodule of M, to show that $ann_R N$ is semimaximal ideal of R, since $N \subseteq M$, which implies that $ann_R M \subseteq ann_R N$. But $ann_R(M)$ is SS-Module. Therefore $ann_R(N)$ is the semimaximal ideal of R by [hatam proposition (1.2.11) p.20].

Hence N is a SS - Module.

Now, we state and prove the following results.

Proposition (2.3)

Zm as a Z-module is SS-Module if and only if m=p1. p2……pn, where pi is a distinct prime number, i=1,2,…..,n.

Proof

Suppose that Z_m is a SS-Module. Then $ann_z Z_m$ is semimaximal ideal of Z, to show that m = p1.p2….p_n, where pi is distinct prime number, i= 1,2,….,n.

$ann_Z Z_m = Mz = \cap_{i=1}^{n}(pi)Z$, where (p_i) is a maximal ideal of Z, for all i=1,2,3,…., n.

$=p_1Z \cap p_2Z \cap \ldots\ldots \cap p_nZ$.

=(p1.p2……pn). Therefore, $m=p_1.p_2\ldots..p_n$, where pi is distinct prime number, i=1,2,3….,n

Conversely, if m= $p_1.p_2\ldots\ldots p_n$, whre p_i is distinct prime number, i= 1,2,3,….,n.

To show that Z_m is a SS-Module , $ann_z Zm=Mz = (p_1.p_2\ldots\ldots p_n)\ Z = p_1Z.p_2Z\ldots..p_nZ$

$=(p_1.p_2\ldots\ldots p_n)=\cap_{i=1}^{n} p_i$.

Hence Z_m is a SS-Module.

The following theorem gives some characterizations for SS-Modules

Theorem (2.4)

Let M be a finitely generated R-module.Then:-

(1) M is a SS - Module.

(2) $(ann_R(M)$: A) is a semimaximal oideal of R for every ideal of A such that $A \not\subseteq ann_z(M)$.

(3) $(ann_R(M)$: r) is the semimaximal ideal of R for every element $r \in R$ such that $r \notin ann_R(M)$.

(4) $ann_R(m)$ is a semimaximal ideal of R, for every non-zero element $m \in M$.

Proof

(1) ⇒ (2) Suppose that M is SS-Module .Then $ann_R(M)$ is the semimaximal ideal of R. Assume that A is an ideal of R such that $A \not\subseteq ann_R(M)$. Since

$ann_R(M) \subseteq (ann_R(M): A)$..

Thus ,by [hatam, propo.(1.2.11), p. 20]

We get ($ann_R(M)$: A) is a semimaximal ideal of R.

(2) ⇒ (3) By taking A=R and from e (2), we get the result.

(3) ⇒ (4) Let 0≠m∈M. Becuse $1 \notin ann_R(m)$, ($ann_R(m)$: R) is semimaximalby (3). But ($ann_R(m)$: R) = $ann_R(m)$, so $ann_R(m)$ is the semimaximal ideal of R.

(4) ⇒ (1) Since M is finitely generated, M=$\sum_{i=1}^{n} Rx_i$, xi∈ M. Thus $ann_R(M) = \cap_{x \in M} ann_R(x)$, by (4), $ann_R(x)$ is semimaximal ideal of R. Thus $\cap_{x \in M} ann_R(x)$ is the semimaximal ideal of R by [hatam, cor (1.2.15), p. 21]. Therefore $ann_R(M)$ Is semimaximal ideal of R. Hence M is a SS - Module.

The following proposition shows a direct sum of SS-Modules is a SS - Module.

Proposition (2.5)

Let M1and M2 be two R-modules .Then $M_1 \oplus M_2$ is a SS-Modules, then by (remarks and example (2.3) (5)), M1and M2 are SS-Module .

Conversely, assume that M_1andM_2 are w SS-Modules, let 0≠m∈M, m=(m_1,m_2) and $ann_R(m) = ann_R(m_1) \cap ann_R(m_2)$, since $ann_R(m_1)$ and $ann_R(m_2)$ are semimaximal ideals of R. Thus $ann_R(m_1) \cap ann_R(m_2)$ is a semimaximal ideal of R [hatam, propo.(1.2.14, p.21]. Then $ann_R(m)$ is the semimaximal ideal of R and hence M=$M_1 \oplus M_2$ is a SS-Module

So, we have the following application of the above proposition

Corollary (2.6):-$\oplus_{\alpha \in \Lambda} M_\alpha$ is a SS-Module for all α.

3. SS-MODULE AND REGULAR MODULES

Proposition (3.1)

If M is a SS-Module, then $\frac{R}{ann_R(M)}$ is the regular ring.

Proof

Since M is SS-Module, then $ann_R(M)$ is the semimaximal ideal of R. Thus, by [hatam, Propo. (1.3.1), p. 26], we get $\frac{R}{ann_R M}$ is the regular ring.

The following corollary is an immediate consequence of the proposition (3.1)

Corollary (3.2)

If 0≠x is an element of an R-module M such that $ann_R(x)$ is semimaximal ideal of R ,then $\frac{R}{ann_R(x)}$ is regular ring.

Proof

It is obvious according to the theorem (2.4) and proposition (3.1).

Proposition (3.3)

Let M be a SS-Module.Then M is a regular R - module.

Proof

Let M be SS-Module, 0≠x∈M. then $ann_R(x)$ is semimax

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